Double parton scattering, diffraction and effective cross section

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The rates of multiparton collisions in high energy hadronic interactions provide information on the typical transverse distances between partons in the hadron structure. The different configurations of the hadron in transverse space are, on the other hand, at the origin of hadron diffraction. The relation between the two phenomena is exploted in an eikonal model of hadronic interactions.

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1. INTRODUCTION

The high parton luminosity and the proximity with the initial runs at the LHC have generated a renewed interest in multiple parton interactions[1]. The rate of multiple parton interactions is in fact estimated to be rather large at the LHC[2][3] giving rise to different effects, among others a substantial increase of the estimated background in channels of interest for the search of new physics (e.g. for Higgs discovery[4]). Unfortunately the process cannot be estimated in a straightforward way, due to the lack of knowledge of the non-perturbative input, namely the multi-parton correlations of the hadron structure. Existing experimental results on multiple parton interactions[5][6][7] show in fact that the simplest implementation of the phenomenon, where multi-parton correlations are neglected, is unable to reproduce the data[8][9].

The unexpected feature of multiple parton interactions, emerged from the experimental studies of the phenomenon, is represented by the small value of the "effective cross section", the scale factor characterizing double parton collisions.

The simplest possibility for multiparton interactions is a Poissonian distribution, at a fixed hadronic impact parameter[10][11][12][13]. The Poissonian at fixed impact parameter takes into account only the most obvious correlation between partons, namely that all partons must be localized inside the volume occupied by the hadron. An argument in favour of this simplifying attitude is that other sources of correlations, e.g. conservation laws, are diluted at small x (where multiple parton collisions are a sizable effect) by the large parton population.

Inclusive and exclusive cross sections in the Poissonian model

In the Poissonian model one introduces the three dimensional parton density $\Gamma(x,b)$, representing the average number of partons with a given momentum fraction x and transverse coordinate b (the dependence on flavor and on the resolution of the process is implicit) and

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one makes the simplifying assumption that the dependence on the transverse and longitudinal degrees of freedom can be factorized as $\Gamma(x,b)=G(x)f(b)$, with $G(x)$ the usual parton distribution function and $f(b)$ a function normalized to one and representing the distributions of partons in transverse space. The inclusive cross section for large p_t parton production may hence be expressed as

$$\begin{aligned}\sigma_S &= \int_{p_t^c} G(x)\hat{\sigma}(x,x')G(x')dxdx' \\ &= \int_{p_t^c} G(x)f(b)\hat{\sigma}(x,x')G(x')f(b-\beta)d^2bdxdx'd^2\beta \end{aligned} \tag{1}$$

where $\hat{\sigma}(x,x')$ is the parton-parton cross section integrated with the cutoff p_t^c, introduced to distinguish hard and soft parton collisions, β is the impact parameter of the hadronic interaction and b is the transverse coordinate of the two colliding partons (the relative transverse distance between the interacting partons is neglected, being here compared with the much larger hadronic dimension).

Disregarding the effects of correlations in the multi-parton distributions, the inclusive cross section for a double parton scattering σ_D is analogously expressed by

$$\begin{aligned}\sigma_D &= \frac{1}{2!}\int_{p_t^c} G(x_1)f(b_1)\hat{\sigma}(x_1,x_1')G(x_1')f(b_1-\beta)d^2b_1dx_1dx_1' \times \\ &\qquad \times G(x_2)f(b_2)\hat{\sigma}(x_2,x_2')G(x_2')f(b_2-\beta)d^2b_2dx_2dx_2'd^2\beta \\ &= \frac{1}{2!}\int\Big(\int_{p_t^c} G(x)f(b)\hat{\sigma}(x,x')G(x')f(b-\beta)d^2bdxdx'\Big)^2 d^2\beta \\ &= \frac{1}{2}\frac{\sigma_S^2}{\sigma_{eff}} \end{aligned} \tag{2}$$

where the scale factor $\sigma_{eff}^{-1}=\int d^2\beta\big[F(\beta)\big]^2$, with $F(\beta)=\int f(b)f(b-\beta)d^2b$, has been introduced[14]. As σ_D is obtained by multiplying σ_S by the ratio σ_S/σ_{eff}, the scale factor σ_{eff} represents the value of σ_S where the inclusive rate of double collisions becomes as large as the inclusive rate of single collisions (apart from the factor 1/2 due to the identity of the two interactions). Similarly the Nth parton scattering inclusive cross section σ_N is given by

$$\begin{aligned}\sigma_N &= \int\frac{1}{N!}\Big(\int_{p_t^c} G(x)f(b)\hat{\sigma}(x,x')G(x')f(b-\beta)d^2bdxdx'\Big)^N d^2\beta \\ &= \int\frac{1}{N!}\big(\sigma_S F(\beta)\big)^N d^2\beta \end{aligned} \tag{3}$$

In the simplest uncorrelated case one may easily give explicit expressions also for the *exclusive* hard cross sections, namely for the various contributions to the inelastic cross section due to the different multiparton scattering processes. The integrand $\frac{1}{N!}\big(\sigma_S F(\beta)\big)^N$ is in fact dimensionless and, once normalized, it may be understood as the probability for a Nth parton collision process. The hard cross section σ_{hard}, namely the contribution to the inelastic cross section given by all events with *at least* one parton collision with momentum transfer greater than the cutoff p_t^c, is hence expressed by

$$\sigma_{hard} = \sum_{N=1}^{\infty}\int d^2\beta \frac{\big(\sigma_S F(\beta)\big)^N}{N!} e^{-\sigma_S F(\beta)} = \int d^2\beta\Big[1 - e^{-\sigma_S F(\beta)}\Big] \tag{4}$$

and

$$\frac{\big(\sigma_S F(\beta)\big)^N}{N!} e^{-\sigma_S F(\beta)} = \mathrm{P}_N(\beta) \tag{5}$$

represents the probability of having N parton collisions in a hadronic interaction at impact parameter β. The relation with the inelastic cross section is

$$\sigma_{inel} = \sigma_{soft} + \sigma_{hard} \tag{6}$$

with σ_{soft} the soft contribution to the inelastic cross section σ_{inel}, the two contributions σ_{soft} and σ_{hard} being defined through the cutoff in the momentum exchanged between partons, p_t^c. Notice that, differently from the case of the inclusive cross sections, which are divergent for $p_t^c \to 0$, both σ_{hard} and all exclusive contributions to σ_{hard}, with a given number of parton collisions, are finite in the infrared limit.

Dispersion and effective cross section

The many-parton inclusive cross sections are proportional to the moments of the distribution in multiple parton collisions.
In particular the single and the double parton inclusive scattering cross sections are proportional to the average number of parton scatterings and to the dispersion of the distribution in the number of collisions. The average number of parton scatterings is in fact given by:

$$\langle N\rangle\sigma_{hard} = \int d^2\beta \sum_{N=1}^{\infty} \frac{N\big[\sigma_S F(\beta)\big]^N}{N!} e^{-\sigma_S F(\beta)} = \int d^2\beta\sigma_S F(\beta) = \sigma_S \tag{7}$$

while for the dispersion one obtains:

$$\begin{aligned}\frac{\langle N(N-1)\rangle}{2}\sigma_{hard} &= \frac{1}{2}\int d^2\beta \sum_{N=2}^{\infty} \frac{N(N-1)\big[\sigma_S F(\beta)\big]^N}{N!} e^{-\sigma_S F(\beta)} \\ &= \frac{1}{2}\int d^2\beta\big[\sigma_S F(\beta)\big]^2 = \sigma_D \end{aligned}\tag{8}$$

The relations between σ_S and $\langle N\rangle$ and between σ_D and $\langle N(N-1)\rangle$ are not peculiar of the simplest Poissonian distribution. Their validity is indeed much more general[15][8]. The same relations can in fact be obtain also when considering the most general case of multiparton distributions, namely including all possible multi-parton correlations (in particular the correlations

induced by conservation laws). On the other hand, the direct link, between inclusive cross sections and moments of the distribution in multiplicity of collisions, gets spoiled when taking into account connected multiparton interactions, namely $3 \rightarrow 3$ etc. parton collision processes, which nevertheless should not give rise to major effects in *pp* collisions even at the LHC energy. Limiting the discussion to the case of hadronic interaction, namely excluding hadron-nucleus and nucleus-nucleus collisions, one may hence write

$$\langle N\rangle\sigma_{hard} = \sigma_S \quad \text{and} \quad \frac{1}{2}\langle N(N-1)\rangle\sigma_{hard} = \sigma_D \tag{9}$$

The effective cross section σ_{eff} is therefore linked to the dispersion $\langle N(N-1)\rangle$ and to the average $\langle N\rangle$ by the relation:

$$\langle N(N-1)\rangle = \langle N\rangle^2\frac{\sigma_{hard}}{\sigma_{eff}} \tag{10}$$

which implies that, if one had a Poissonian distribution of multiple parton collisions also after integration over the impact parameter β, one would have $\sigma_{eff} = \sigma_{hard}$. Even in the simplest case, the distribution is however Poissonian before integration on the impact parameter. The final distribution in the number of parton collisions will hence have a larger dispersion as compared with the Poissonian, which implies σ_{eff} smaller than σ_{hard}.
The actual values of σ_{eff} and of σ_{hard} depend on the functional form used for $F(\beta)$. For $F(\beta) = \exp(-\beta^2/R^2)/\pi R^2$ one obtains:

$$\sigma_{hard} = 2\pi R^2\big[\gamma + \ln\kappa + E_1(\kappa)\big] \tag{11}$$

where $\gamma = 0.5772\ldots$ is Euler's constant, $\kappa = \sigma_S/(2\pi R^2)$ and $E_1(x)$ is the exponential integral. For κ small $\sigma_{hard} \rightarrow 2\pi R^2\kappa = \sigma_S$, while for κ large (namely $\sigma_S \rightarrow \infty$) one obtains $\sigma_{hard} \rightarrow 2\pi R^2\big(\gamma + ln\kappa\big)$. Here $\sigma_{eff} = 2\pi R^2$. The value of σ_{hard} is therefore proportional to the value of σ_{eff}, the proportionality factor being slightly dependent on energy and cutoff. Sensible values of the hadron-hadron c.m. energy and of the cutoff give values of σ_{hard} $30 - 40\%$ larger than σ_{eff}. Asymptotically (at high energy and fixed momentum cutoff) one expects $\sigma_{hard} \approx \sigma_{inel}$. The simplest expectation is hence that σ_{eff} should not be much smaller than σ_{inel}.

On the contrary the experimental indication[5][6][7] is that σ_{eff} may be more than a factor three smaller than σ_{inel}. Although one may obtain a value of σ_{eff} sizably smaller than σ_{inel} choosing appropriate analytic forms for $F(\beta)$[16], sensible choices of $F(\beta)$ give results qualitatively similar to the simplest gaussian case[8].

An obvious possibility to increase the dispersion of the distribution in the number of collisions, namely to obtain smaller values of σ_{eff}, is to consider a different distribution in the number of collisions at a fixed impact parameter, which evidently implies a non secondary role of correlations in the multiparton distribution. Including correlations one may hence easily obtain a large dispersion in the final distribution of multiple parton interactions and, correspondingly, a small value of σ_{eff}[17].

A source of correlation for the parton population is the fluctuation of the hadron in its transverse dimension, which is a phenomenon related directly to hadronic diffraction. Although

the diffractive cross section is expected to be small at high energies, recent estimates indicate sizably large effects of diffraction in pp collisions also at the LHC[18][19][20][21].

The purpose of the present paper is to study the link between diffraction and multiparton collisions. The dispersion in the collisions distribution, and hence of the expected value of the effective cross section, will thus be derived in the simplest multichannel eikonal model of high energy hadronic interactions, capable of reproducing the total, elastic, inelastic, single and double diffractive cross sections in high energy pp collisions.

To make the argument self sufficient, the multi-channel eikonal model will be discussed in detail in the next section. The cross sections of inclusive and exclusive hard processes will hence be derived in the following paragraph. The effective cross section, at TeVatron and LHC energies, will be finally estimated in the simplest two-channel case, using as input the parameters fitted to reproduce the available information on soft cross sections in high energy pp collisions.

2. THE MULTI-CHANNEL EIKONAL MODEL

Let's represent with ψ_μ the initial hadron and its diffractive states and with ϕ_η the eigenstates of the forward T matrix scattering operator and let's normalize the imaginary part of the nucleon-nucleon forward T matrix operator to the total cross section of the different hadronic channels $\mu\ \nu$, $[\sigma_{tot}]_{\mu\nu}$:

$$\mathrm{Im}\langle\psi_\mu\psi_\nu|T|\psi_\mu\psi_\nu\rangle = [\sigma_{tot}]_{\mu\nu} \tag{12}$$

here and in the following the indices μ and ν will refer either to the hadron or to its various diffractive states, while the indices η and ρ to the eigenstates of the forward T matrix scattering operator. One may hence write:

$$\mathrm{Im}\langle\psi_\mu\psi_\nu|T|\psi_\mu\psi_\nu\rangle = \sum_{\eta\rho}|a_{\mu\eta}|^2|a_{\nu\rho}|^2\mathrm{Im}T_{\eta\rho} \tag{13}$$

where $T_{\eta\rho} = \langle\phi_\eta\phi_\rho|T|\phi_\eta\phi_\rho\rangle$ and $a_{\mu\eta} = \langle\psi_\mu|\phi_\eta\rangle$.
Let's denote with $t_{\eta\rho}$ the one-Pomeron exchange forward scattering amplitude of the states ϕ_η and ϕ_ρ, with imaginary part normalized to the cross section $\sigma_{\eta\rho}$.
In the case of interest large numbers of Pomerons may be exchanged.
Assuming independency for the different exchanges, one may write:

$$T_{\eta\rho} = \sum_{k=1}^{\infty}\langle\phi_\eta\phi_\rho|t_k|\phi_\eta\phi_\rho\rangle \qquad \text{where} \qquad \frac{\mathrm{i}}{2}\langle\phi_\eta\phi_\rho|t_k|\phi_\eta\phi_\rho\rangle = \int d^2b\frac{1}{k!}\Big(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\Big)^k \tag{14}$$

and

$$\frac{\mathrm{i}}{2}T_{\eta\rho} = \int d^2b\sum_{k=1}^{\infty}\frac{1}{k!}\Big(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\Big)^k = \int d^2b\Big\{\mathrm{e}^{\frac{\mathrm{i}t_{\eta\rho}(b)}{2}}-1\Big\} \tag{15}$$

The total cross section between the eigenstates of the T matrix ϕ_η and ϕ_ρ is obtained through the optical theorem:

$$\sigma^{\eta\rho}_{tot} = 2\mathrm{Re}\int d^2b\Big\{1-\mathrm{e}^{\frac{\mathrm{i}t_{\eta\rho}(b)}{2}}\Big\} \tag{16}$$

which may be also expressed as a sum of multiple collisions amplitudes:

$$\sigma^{\eta\rho}_{tot} = 2\mathrm{Re}\int d^2b\sum_{k=1}^{\infty}\frac{1}{k!}\left(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\right)^k \mathrm{e}^{-\frac{\mathrm{i}t_{\eta\rho}}{2}} \tag{17}$$

the total cross sections between the hadronic states ψ_μ and ψ_ν is hence given by

$$[\sigma_{tot}]_{\mu\nu} = \sum_{\eta\rho}|a_{\mu\eta}|^2|a_{\nu\rho}|^2\sigma^{\eta\rho}_{tot} \tag{18}$$

The different (leading) contributions to the cross section due to the various elastic, diffractive and production channels are readily derived[22]. Let's first consider the contributions to the cuts due to the scattering between the eigenstates of the the T matrix ϕ_η and ϕ_ρ, $T_{\eta\rho}$. Each $t_{\eta\rho}$ represents a single Pomeron exchange; let's denote with $D^{\eta\rho}_{k,m}$ the contribution to the inelastic cross section where k Pomerons are exchanged and m Pomerons are cut, $k \geq m$. From Eq.(15) one obtains:

$$D^{\eta\rho}_{k,m} = \int d^2b\frac{1}{k!}\binom{k}{m}\sum_{l=0}^{k-m}\binom{k-m}{l}\left[\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\right]^l\left[\left(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\right)^*\right]^{k-m-l} \times \sigma_{\eta\rho}(b)^m \tag{19}$$

- a factor $\mathrm{i}t_{\eta\rho}(b)/2$ is associated to each Pomeron on the left hand side of the cut,

- a factor $(\mathrm{i}t_{\eta\rho}(b)/2)^*$ is associated to each Pomeron on the right hand side of the cut

- a factor $2\mathrm{Im}(\mathrm{i}t_{\eta\rho}(b)/2) = \sigma_{\eta\rho}(b)$ is associated to each cut-Pomeron.

The sum over l represents all possibilities, namely l Pomerons on the left hand side of the cut and $k-m-l$ Pomerons on the right hand side of the cut.
The sum over l can be performed giving:

$$\begin{aligned} D^{\eta\rho}_{k,m} &= \int d^2b\frac{1}{k!}\binom{k}{m}\left[\frac{\mathrm{i}t_{\eta\rho}(b)}{2}+\left(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\right)^*\right]^{k-m} \times \sigma_{\eta\rho}(b)^m \\ &= \int d^2b\frac{1}{k!}\binom{k}{m}\big(-\sigma_{\eta\rho}(b)\big)^{k-m} \times \sigma_{\eta\rho}(b)^m \end{aligned} \tag{20}$$

where the relation $\mathrm{i}t_{\eta\rho}(b)/2 + (\mathrm{i}t_{\eta\rho}(b)/2)^* = -\sigma_{\eta\rho}(b)$ has been used.
The contribution to the cross section $\sigma^{\eta\rho}_m$, corresponding to m cut-Pomerons, is obtained by summing over all elastic scatterings in $D^{\eta\rho}_{k,m}$:

$$\sigma_m^{\eta\rho} = \sum_{k=m}^{\infty} D_{k,m}^{\eta\rho} = \int d^2b \sum_{k=m}^{\infty} \frac{1}{k!} \frac{k!}{(k-m)!m!} \big(-\sigma_{\eta\rho}(b)\big)^{k-m} \sigma_{\eta\rho}(b)^m$$
$$= \int d^2b \frac{1}{m!} \big(\sigma_{\eta\rho}(b)\big)^m \mathrm{e}^{-\sigma_{\eta\rho}(b)} \tag{21}$$

The contribution to the inelastic cross sections due to the scattering between the eigenstates of the the T matrix ϕ_η and ϕ_ρ is the result of summing all $\sigma_m^{\eta\rho}$'s:

$$\sigma_{in}^{\eta\rho} = \sum_{m=1}^{\infty} \sigma_m^{\eta\rho} = \int d^2b \sum_{m=1}^{\infty} \frac{1}{m!} \big(\sigma_{\eta\rho}(b)\big)^m \mathrm{e}^{-\sigma_{\eta\rho}(b)} = \int d^2b \Big\{ 1 - \mathrm{e}^{-\sigma_{\eta\rho}(b)} \Big\} \tag{22}$$

The inelastic cross sections of the hadronic states ψ_μ and ψ_ν is hence given by

$$[\sigma_{in}]_{\mu\nu} = \sum_{\eta\rho} |a_{\mu\eta}|^2 |a_{\nu\rho}|^2 \sigma_{in}^{\eta\rho} \tag{23}$$

The elastic and diffractive cuts of the scattering amplitude of the hadronic states ψ_μ and ψ_ν are obtained from the elastic cuts of the scattering between the eigenstates of the T matrix ϕ_η and ϕ_ρ (of course the only possibilities for the eigenstates of the T matrix is to be either absorbed or to scatter elastically).

One my obtain the elastic cross section of the scattering of ϕ_η and ϕ_ρ working out all elastic cuts between the exchanged Pomerons. Let's introduce $E_k^{\eta\rho}$, the contribution to the elastic scattering cross section of the eigenstates ϕ_η and ϕ_ρ with k Pomerons exchanged. Obviously elastic cuts are possible only for $k \geq 2$:

$$E_k^{\eta\rho} = \int d^2b \frac{1}{k!} \Big\{ \sum_{l=1}^{k-1} \binom{k}{l} \Big[\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\Big]^l \Big[\Big(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\Big)^*\Big]^{k-l} \Big\}$$
$$= \int d^2b \frac{1}{k!} \Big\{ \Big[\frac{\mathrm{i}t_{\eta\rho}(b)}{2} + \Big(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\Big)^*\Big]^k - \Big[\Big(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\Big)^*\Big]^k - \Big[\Big(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\Big)\Big]^k \Big\}$$
$$= \int d^2b \frac{1}{k!} \Big\{ \big(-\sigma_{\eta\rho}(b)\big)^k - 2 \times \mathrm{Re}\Big(-\frac{\sigma_{\eta\rho}(b)}{2}(1 - \mathrm{i}\alpha_{\eta\rho})\Big)^k \Big\} \tag{24}$$

The contribution due to the elastic scattering between the eigenstates of the the T matrix ϕ_η and ϕ_ρ is obtained by summing over all elastic cuts:

$$\sigma_{el}^{\eta\rho} = \sum_{k=2}^{\infty} E_k^{\eta\rho} = \int d^2b \Big\{ \mathrm{e}^{-\sigma_{\eta\rho}(b)} - 1 + \big(-\sigma_{\eta\rho}(b)\big)$$
$$-2 \times \mathrm{Re}\Big[\mathrm{e}^{-\frac{\sigma_{\eta\rho}(b)}{2}(1-\mathrm{i}\alpha_{\eta\rho})} - 1 + \Big(-\frac{\sigma_{\eta\rho}(b)}{2}(1 - \mathrm{i}\alpha_{\eta\rho})\Big)\Big]\Big\}$$
$$= \int d^2b \Big| 1 - \mathrm{e}^{-\frac{\sigma_{\eta\rho}(b)}{2}(1-\mathrm{i}\alpha_{\eta\rho})} \Big|^2 \tag{25}$$

The total cross section of the scattering between the eigenstates of the the T matrix ϕ_η and ϕ_ρ results from the sum of the elastic and inelastic contributions:

$$\begin{aligned}
\sigma_{el}^{\eta\rho} + \sigma_{in}^{\eta\rho} &= \int d^2b \Big\{ 1 - 2 \times \mathrm{Re}\Big[\mathrm{e}^{-\frac{\sigma_{\eta\rho}(b)}{2}(1-\mathrm{i}\alpha_{\eta\rho})} \Big] + \mathrm{e}^{-\sigma_{\eta\rho}(b)} + 1 - \mathrm{e}^{-\sigma_{\eta\rho}(b)} \Big\} \\
&= 2 \int d^2b \Big\{ \mathrm{Re}\Big[1 - \mathrm{e}^{-\frac{\sigma_{\eta\rho}(b)}{2}(1-\mathrm{i}\alpha_{\eta\rho})} \Big] \Big\} = \sigma_{tot}^{\eta\rho} \qquad (26)
\end{aligned}$$

which is precisely the expression for $\sigma_{tot}^{\eta\rho}$ in Eq.16.
The total and inelastic cross sections between the hadronic states ψ_μ and ψ_ν are hence given by

$$[\sigma_{tot}]_{\mu\nu} = \sum_{\eta\rho} |a_{\mu\eta}|^2 |a_{\nu\rho}|^2 \sigma_{tot}^{\eta\rho} \qquad (27)$$

$$[\sigma_{in}]_{\mu\nu} = \sum_{\eta\rho} |a_{\mu\eta}|^2 |a_{\nu\rho}|^2 \sigma_{in}^{\eta\rho} \qquad (28)$$

The sum of the elastic (el), single diffractive (SD) and double diffractive (DD) cross sections in hh collisions is easily obtained:

$$\begin{aligned}
\sigma_{el} + \sigma_{SD} + \sigma_{DD} = \sum_{\eta\rho\gamma\sigma} \langle \psi_h \psi_h | \phi_\eta \phi_\rho \rangle \; \times \; \langle \phi_\eta \phi_\rho | T | \phi_\eta \phi_\rho \rangle \langle \phi_\eta \phi_\rho | \sum_{\mu\nu} | \psi_\mu \psi_\nu \rangle \langle \psi_\mu \psi_\nu | \\
\times | \phi_\gamma \phi_\sigma \rangle \langle \phi_\gamma \phi_\sigma | T^\dagger | \phi_\gamma \phi_\sigma \rangle \langle \phi_\gamma \phi_\sigma | \psi_h \psi_h \rangle \qquad (29)
\end{aligned}$$

The sum over the final states $\mu\nu$ gives one for completeness and one is left with the product

$$\langle \phi_\eta \phi_\rho | \phi_\gamma \phi_\sigma \rangle = \delta_{\eta\gamma} \delta_{\rho\sigma} \qquad (30)$$

One hence obtains

$$\begin{aligned}
\sigma_{el} + \sigma_{SD} + \sigma_{DD} &= \sum_{\eta\rho} \big| \langle \psi_h \psi_h | \phi_\eta \phi_\rho \rangle \big|^2 \big| \langle \phi_\eta \phi_\rho | T | \phi_\eta \phi_\rho \rangle \big|^2 \\
&= \sum_{\eta\rho} |a_{h\eta}|^2 |a_{h\rho}|^2 \sigma_{el}^{\eta\rho} \qquad (31)
\end{aligned}$$

since

$$\langle \psi_h \psi_h | \phi_\eta \phi_\rho \rangle = a_{h\eta} a_{h\rho} \qquad \text{and} \qquad \big| \langle \phi_\eta \phi_\rho | T | \phi_\eta \phi_\rho \rangle \big|^2 = \sigma_{el}^{\eta\rho} \qquad (32)$$

and summing the inelastic cross section

$$\begin{aligned}
\sigma_{el} + \sigma_{SD} + \sigma_{DD} + \sigma_{in} &= \sum_{\eta\rho} |a_{h\eta}|^2 |a_{h\rho}|^2 \big(\sigma_{el}^{\eta\rho} + \sigma_{in}^{\eta\rho} \big) \\
&= \sum_{\eta\rho} |a_{h\eta}|^2 |a_{h\rho}|^2 \sigma_{tot}^{\eta\rho} = \sigma_{tot} \qquad (33)
\end{aligned}$$

The result shows that the model is manifestly consistent with unitarity, as the total cross section, obtained through the optical theorem, is explicitly given by the sum of all possible intermediate states obtained cutting the forward amplitude.

In the gaussian case

$$\sigma_{\eta\rho}(b)=\nu_{\eta\rho}(s)\frac{\exp[-b^2/R^2_{\eta\rho}(s)]}{\pi R^2_{\eta\rho}(s)} \tag{34}$$

the cross sections are expressed by close analytic forms:

$$\begin{aligned}
\sigma^{\eta\rho}_{el} &= \pi R^2_{\eta\rho}(s)\Big\{2\mathrm{Re}\big[\gamma+\ln\kappa_{\eta\rho}+E_1(\kappa_{\eta\rho})\big]-\big[\gamma+\ln\kappa'_{\eta\rho}+E_1(\kappa'_{\eta\rho})\big]\Big\}\\
\sigma^{\eta\rho}_{T} &= 2\pi R^2_{\eta\rho}(s)\mathrm{Re}\big[\gamma+\ln\kappa_{\eta\rho}+E_1(\kappa_{\eta\rho})\big]\\
\sigma^{\eta\rho}_{in} &= \pi R^2_{\eta\rho}(s)\big[\gamma+\ln\kappa'_{\eta\rho}+E_1(\kappa'_{\eta\rho})\big]
\end{aligned}$$

where γ is Euler's constant,

$$\kappa_{\eta\rho}=\frac{\nu_{\eta\rho}(s)}{\pi R^2_{\eta\rho}(s)}\frac{1-\mathrm{i}\alpha_{\eta\rho}}{2},\qquad \kappa'_{\eta\rho}=\frac{\nu_{\eta\rho}(s)}{\pi R^2_{\eta\rho}(s)} \tag{35}$$

and $E_1(x)$ the exponential integral.

Explicit expressions for the hadron-hadron total, elastic, single and double diffractive cross sections at different energies may hence be obtained.

Hard and soft Pomerons

A cut-Pomeron may be either soft, when there are no large p_t partons in the final state, or hard, when large p_t partons are present. The two contributions are conveniently discussed with the help of the eigenstates of the forward T matrix. One may write

$$\sigma_{\eta\rho}=\sigma^S_{\eta\rho}+\sigma^J_{\eta\rho} \tag{36}$$

The contributions to $D^{\eta\rho}_{k,m}$ due to soft and hard Pomeron exchanges are derived by expanding the powers of $\sigma_{\eta\rho}$ in Eq.20:

$$\begin{aligned}
D^{\eta\rho}_{k,m} &= \int d^2b\frac{1}{k!}\binom{k}{m}\left[\frac{\mathrm{i}t_{\eta\rho}(b)}{2}+\left(\frac{\mathrm{i}t_{\eta\rho}(b)}{2}\right)^*\right]^{k-m}\times[\sigma^S_{\eta\rho}(b)+\sigma^J_{\eta\rho}(b)]^m\\
&= \sum_{n=0}^{m}\int d^2b\frac{1}{k!}\binom{k}{m}\binom{m}{n}\big(-\sigma_{\eta\rho}(b)\big)^{k-m}\times[\sigma^J_{\eta\rho}(b)]^n[\sigma^S_{\eta\rho}(b)]^{m-n}=\sum_{n=0}^{m}D^{\eta\rho(n)}_{k,m}
\end{aligned} \tag{37}$$

The contribution to the inelastic cross section, of the eigenstates of the T matrix ϕ_η and ϕ_ρ, $\sigma^{\eta\rho}_{in}$, with k Pomerons exchanged and m Pomerons cut ($m\le k$), where one distinguishes between n hard ($n\le m$) and $m-n$ soft Pomerons, is represented by $D^{\eta\rho(n)}_{k,m}$.

The cross section $\sigma_{hard}^{\eta\rho(n)}$, corresponding to the case of n hard cut-Pomerons, where all soft cut-Pomerons and all elastic scatterings have been summed, is expressed by

$$\begin{aligned}\sigma_{hard}^{\eta\rho(n)} = \sum_{k=n}^{\infty}\sum_{m=n}^{k} D_{k,m}^{\eta\rho} &= \int d^2b \sum_{k=n}^{\infty} \frac{1}{n!(k-n)!}[\sigma_{\eta\rho}^{J}(b)]^n \big[\sigma_{\eta\rho}^{S}(b) - \sigma_{\eta\rho}(b)\big]^{k-n} \\ &= \int d^2b \frac{1}{n!}\big(\sigma_{\eta\rho}^{J}(b)\big)^n \mathrm{e}^{-\sigma_{\eta\rho}^{J}(b)} \end{aligned} \tag{38}$$

the cross section $\sigma_{hard}^{\eta\rho}$, including all events with hard cut-Pomerons, is obtained after summing all terms $\sigma_{hard}^{\eta\rho(n)}$ with $n > 0$:

$$\sigma_{hard}^{\eta\rho} = \sum_{n=1}^{\infty} \int d^2b \frac{1}{n!}\big(\sigma_{\eta\rho}^{J}(b)\big)^n \mathrm{e}^{-\sigma_{\eta\rho}^{J}(b)} = \int d^2b \Big\{1 - \mathrm{e}^{-\sigma_{\eta\rho}^{J}(b)}\Big\} \tag{39}$$

The cross sections $\sigma_{tot}^{\eta\rho}$, $\sigma_{in}^{\eta\rho}$ and $\sigma_{hard}^{\eta\rho}$ are all expressed by Poissonians at fixed impact parameter: $\sigma_{tot}^{\eta\rho}$ is a superposition of uncut-Pomeron amplitudes, $\sigma_{in}^{\eta\rho}$ is a superposition of cut-Pomerons and $\sigma_{hard}^{\eta\rho}$ is a superposition of hard cut-Pomerons. In particular only hard processes contribute to the shadowing corrections of $\sigma_{hard}^{\eta\rho}$, which hence belongs to the category of the self-shadowing cross sections[23].

Soft cross section

Let's consider $D_{k,m}^{\eta\rho(0)}$, namely the contribution to the inelastic cross section of the states ϕ_η and ϕ_ρ with k Pomerons exchanged and m soft cut-Pomeron ($m \leq k$)

$$D_{k,m}^{\eta\rho(0)} = \int d^2b \frac{1}{k!}\binom{k}{m}\big[-\sigma_{\eta\rho}(b)\big]^{k-m}\big[\sigma_{\eta\rho}^{S}(b)\big]^m \tag{40}$$

The contribution to the soft inelastic cross section, with m soft cut-Pomeron $\sigma_m^{\eta\rho(0)}$, is obtained summing all elastic scatterings in $D_{k,m}^{\eta\rho(0)}$:

$$\begin{aligned}\sigma_m^{\eta\rho(0)} &= \sum_{k=m}^{\infty} D_{k,m}^{\eta\rho(0)} = \int d^2b \sum_{k=m}^{\infty} \frac{1}{m!(k-m)!}\big[-\sigma_{\eta\rho}(b)\big]^{k-m}\big[\sigma_{\eta\rho}^{S}(b)\big]^m \\ &= \int d^2b \frac{1}{m!}\big[\sigma_{\eta\rho}^{S}(b)\big]^m \sum_{l=0}^{\infty} \frac{\big[-\sigma_{\eta\rho}(b)\big]^l}{l!} \\ &= \int d^2b \frac{1}{m!}\big[\sigma_{\eta\rho}^{S}(b)\big]^m \mathrm{e}^{-\sigma_{\eta\rho}(b)} \end{aligned} \tag{41}$$

The soft cross section $\sigma_{soft}^{\eta\rho}$, including all soft cut-Pomerons and all additional elastic scatterings, is hence given by

$$\sigma_{soft}^{\eta\rho} = \sum_{m=1}^{\infty} \sigma_m^{\eta\rho(0)}$$

$$
\begin{aligned}
&= \int d^2b \Big\{ \mathrm{e}^{\sigma^S_{\eta\rho}(b)-\sigma_{\eta\rho}(b)} - \mathrm{e}^{-\sigma_{\eta\rho}(b)} \Big\} \\
&= \int d^2b \Big\{ \mathrm{e}^{-\sigma^J_{\eta\rho}(b)} - \mathrm{e}^{-\sigma_{\eta\rho}(b)} \Big\} \\
&= \quad \sigma^{\eta\rho}_{in} - \sigma^{\eta\rho}_{hard}
\end{aligned}
\tag{42}
$$

The sum of all intermediate states, generated by cut-Pomerons between eigenstates of the T matrix, gives the inelastic cross section; the sum over all intermediate states, generated without cutting Pomerons, gives the elastic cross section and the sum of the two different contributions gives the total cross section.
When considering hard cut-Pomerons, the sum over all intermediate states gives the hard cross section; the sum over all intermediate states, generated without hard cut-Pomerons, gives the soft cross section plus the elastic cross section and the sum of the two different contributions gives the total cross section.

The hard and soft cross sections between the hadronic states μ and ν are therefore

$$
\begin{aligned}
(\sigma_{hard})_{\mu\nu} &= \sum_{\eta\rho} |a_{h\eta}|^2 |a_{h\rho}|^2 \sigma^{\eta\rho}_{hard} \\
(\sigma_{soft})_{\mu\nu} &= \sum_{\eta\rho} |a_{h\eta}|^2 |a_{h\rho}|^2 \sigma^{\eta\rho}_{soft} = (\sigma_{in})_{\mu\nu} - (\sigma_{hard})_{\mu\nu}
\end{aligned}
\tag{43}
$$

3. AVERAGE, DISPERSION AND INCLUSIVE CROSS SECTIONS

Let's now go to the case of interest, where the initial hadron state is the proton. The indices μ, ν may hence be dropped. The inelastic pp cross sections is

$$
\begin{aligned}
\sigma_{in} = \sum_{\eta\rho} |a_\eta|^2 |a_\rho|^2 \sigma^{\eta\rho}_{in} &= \sum_{\eta\rho} |a_\eta|^2 |a_\rho|^2 \int d^2b \sum_{m=1}^{\infty} \frac{1}{m!} \big[\sigma_{\eta\rho}(b)\big]^m \mathrm{e}^{-\sigma_{\eta\rho}(b)} \\
&= \sum_{\eta\rho} |a_\eta|^2 |a_\rho|^2 \int d^2b \Big\{ 1 - \mathrm{e}^{-\sigma_{\eta\rho}(b)} \Big\}
\end{aligned}
\tag{44}
$$

and the hard cross section is analogously expressed by

$$
\begin{aligned}
\sigma_{hard} = \sum_{\eta\rho} |a_\eta|^2 |a_\rho|^2 \sigma^{\eta\rho}_{hard} &= \sum_{\eta\rho} |a_\eta|^2 |a_\rho|^2 \int d^2b \sum_{m=1}^{\infty} \frac{1}{m!} \big[\sigma^J_{\eta\rho}(b)\big]^m \mathrm{e}^{-\sigma^J_{\eta\rho}(b)} \\
&= \sum_{\eta\rho} |a_\eta|^2 |a_\rho|^2 \int d^2b \Big\{ 1 - \mathrm{e}^{-\sigma^J_{\eta\rho}(b)} \Big\}
\end{aligned}
\tag{45}
$$

while the soft cross section is given by the difference

$$
\sigma_{soft} = \sigma_{in} - \sigma_{hard} = \sum_{\eta\rho} |a_\eta|^2 |a_\rho|^2 \int d^2b \Big\{ \mathrm{e}^{-\sigma^J_{\eta\rho}(b)} - \mathrm{e}^{-\sigma_{\eta\rho}(b)} \Big\}
\tag{46}
$$

which shows that the soft cross section is a border effect and goes to zero in the high energy-fixed momentum exchange limit.

The average number of inelastic collisions is given by the single Pomeron exchange term between the different eigenstates of the T matrix

$$\begin{aligned}\langle m\rangle\sigma_{in} &= \sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\sum_{m=1}^{\infty}\frac{m}{m!}\big[\sigma_{\eta\rho}(b)\big]^m \mathrm{e}^{-\sigma_{\eta\rho}(b)}\\ &= \sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\sigma_{\eta\rho}(b)=\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\nu_{\eta\rho}(s)\end{aligned}\tag{47}$$

and analogously the average number of hard collisions is given by the single scattering inclusive cross section of the perturbative QCD parton model

$$\begin{aligned}\langle m\rangle\sigma_{hard} &= \sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\sum_{m=1}^{\infty}\frac{m}{m!}\big[\sigma^J_{\eta\rho}(b)\big]^m \mathrm{e}^{-\sigma^J_{\eta\rho}(b)}\\ &= \sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\sigma^J_{\eta\rho}(b)\\ &= \sigma_{QCD}(s,p_{cut})|_{single}\end{aligned}\tag{48}$$

The double scattering inclusive cross sections are given by the dispersion of the multiplicity distribution in the number of collisions:

$$\begin{aligned}\frac{1}{2}\langle m(m-1)\rangle\sigma_{in} &= \frac{1}{2}\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\sum_{m=1}^{\infty}\frac{m(m-1)}{m!}\big[\sigma_{\eta\rho}(b)\big]^m \mathrm{e}^{-\sigma_{\eta\rho}(b)}\\ &= \frac{1}{2}\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\big[\sigma_{\eta\rho}(b)\big]^2=\frac{1}{2}\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\frac{\nu^2_{\eta\rho}(s)}{2\pi R^2_{\eta\rho}(s)}\end{aligned}\tag{49}$$

and

$$\begin{aligned}\frac{1}{2}\langle m(m-1)\rangle\sigma_{hard} &= \frac{1}{2}\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\sum_{m=1}^{\infty}\frac{m(m-1)}{m!}\big[\sigma^J_{\eta\rho}(b)\big]^m \mathrm{e}^{-\sigma^J_{\eta\rho}(b)}\\ &= \frac{1}{2}\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\int d^2b\big[\sigma^J_{\eta\rho}(b)\big]^2\\ &= \sigma_{QCD}(s,p_{cut})|_{double}=\frac{1}{2}\frac{\big[\sigma_{QCD}(s,p_{cut})|_{single}\big]^2}{\sigma_{eff}}\end{aligned}\tag{50}$$

where the scale factor σ_{eff}, characterizing double parton interaction processes has been introduced. An analogous scale factor may be introduced for double Pomeron exchanges:

$$\sigma_{eff,P}\equiv\frac{\Big[\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\nu_{\eta\rho}(s)\Big]^2}{\sum_{\eta\rho}|a_\eta|^2|a_\rho|^2\frac{\nu^2_{\eta\rho}(s)}{2\pi R^2_{\eta\rho}(s)}}\tag{51}$$

Introducing the couplings of the Pomeron to the eigenstate of the T matrix η and ρ, g_η and g_ρ, and the Pomeron trajectory

$$\alpha_P(t) = 1 + \Delta_P + \alpha'_P t \tag{52}$$

one has

$$\nu_{\eta\rho}(s) = g_\eta g_\rho \Big(\frac{s}{s_0}\Big)^{\Delta_P} \tag{53}$$

and the scale factor for double Pomeron exchanges, $\sigma_{eff,P}$, is given by

$$\sigma_{eff,P} \equiv \frac{\Big[\sum_{\eta\rho} |a_{\mu\eta}|^2 |a_{\nu\rho}|^2 g_\eta g_\rho\Big]^2}{\sum_{\eta\rho} |a_{\mu\eta}|^2 |a_{\nu\rho}|^2 \frac{g_\eta^2 g_\rho^2}{2\pi R_{\eta\rho}^2(s)}} \tag{54}$$

The scale factor $\sigma_{eff,P}$ has a very smooth energy dependence, in fact it depends on s only logarithmically (through $R_{\eta\rho}(s)$). Similar properties hold for σ_{eff} which, although related to a hard process, is a cutoff independent quantity. In the high energy (and fixed cutoff) limit the inelastic cross section will be saturated by the hard cross section. One may hence argue that, in the regime were the soft component of the interaction becomes negligible, $\sigma_{eff,P}$ and σ_{eff} will coincide. σ_{eff} however should not depend on energy and cutoff (apart from the logarithmic dependence of the hadron radius on s) which brings to the conclusion that if $\sigma_{eff,P}$ and σ_{eff} coincide in the regime were the soft component of the interaction becomes negligible, the two effective cross sections will coincide at any regime.

The simplest two channel case

The simplest two channel case has been studied in detail in the literature and it was shown to be able to reproduce with accuracy available data of total, elastic, single and double diffraction cross sections of pp collisions at different energies[19][20][21]. In the two channel formalism the states observed are either the initial hadron (ψ_h) or its diffractive states, represented globally by a single channel (ψ_D). The eigenstates of the forward interaction operator T are ϕ_1 and ϕ_2. One may hence write

$$\psi_h = \alpha\phi_1 + \beta\phi_2 \tag{55}$$

while ψ_D is represented by the orthogonal superposition

$$\psi_D = -\beta\phi_1 + \alpha\phi_2 \tag{56}$$

with $\alpha^2 + \beta^2 = 1$ (hence the relation with the coefficients $a_{\mu\eta}$ introduced in the previous paragraphs is $a_{11} = a_{22} = \alpha$, $a_{12} = -a_{21} = \beta$).
The total and inelastic hadron-hadron cross sections are

$$\sigma_{tot} = \sigma_{tot}^{11}\alpha^4 + \sigma_{tot}^{12}\alpha^2\beta^2 + \sigma_{tot}^{21}\alpha^2\beta^2 + \sigma_{tot}^{22}\beta^4 \tag{57}$$

$$\sigma_{in} = \sigma_{in}^{11}\alpha^4 + \sigma_{in}^{12}\alpha^2\beta^2 + \sigma_{in}^{21}\alpha^2\beta^2 + \sigma_{in}^{22}\beta^4 \tag{58}$$

and

$$\sigma_{el} + \sigma_{SD} + \sigma_{DD} = \sigma_{el}^{11}\alpha^4 + 2\sigma_{el}^{12}(\alpha\beta)^2 + \sigma_{el}^{22}\beta^4 \tag{59}$$

Unitarity is satisfied, as it may be checked summing all contributions

$$\begin{aligned}\sigma_{el} + \sigma_{SD} + \sigma_{DD} + \sigma_{in} &= \sigma_{el}^{11}\alpha^4 + 2\sigma_{el}^{12}(\alpha\beta)^2 + \sigma_{el}^{22}\beta^4 \\ &\quad +\sigma_{in}^{11}\alpha^4 + \sigma_{in}^{12}\alpha^2\beta^2 + \sigma_{in}^{21}\alpha^2\beta^2 + \sigma_{in}^{22}\beta^4 \\ &= \sigma_{tot}^{11}\alpha^4 + \sigma_{tot}^{12}\alpha^2\beta^2 + \sigma_{tot}^{21}\alpha^2\beta^2 + \sigma_{tot}^{22}\beta^4 = \sigma_{tot}\end{aligned} \tag{60}$$

The most appropriate choice of parameters for reproducing the total, elastic, single and double diffraction cross sections is the following[21]

$$\begin{aligned}\Delta_P &= 0.15, \quad \alpha^{'} = 0.173GeV^{-2}, \quad s_0 = 1GeV^2, \\ \sigma_{11}^0 &= 9.22GeV^{-2}, \quad \sigma_{22}^0 = 3503.5GeV^{-2}, \quad \sigma_{12}^0 = 6.53GeV^{-2}, \\ \sigma_{i,k}^0 &= g_i g_k, \quad \beta = 0.776, \quad R_{0,1}^2 = 10.42GeV^{-2}, \quad r_0^2 = 0.5GeV^{-2}\end{aligned}$$

where

$$\begin{aligned}R_{11}^2(s) &= 2R_{0,1}^2 + r_0^2 + 4\alpha'\ln(s/s_0) \\ R_{12}^2(s) &= R_{0,1}^2 + r_0^2 + 4\alpha'\ln(s/s_0) \\ R_{22}^2(s) &= r_0^2 + 4\alpha'\ln(s/s_0)\end{aligned}$$

With this input one obtains

$$\begin{aligned}\sqrt(s) &= 14\text{TeV} \qquad \sigma_{tot} = 114\text{mb} \qquad \sigma_{inel} = 71\text{mb} \qquad \sigma_{eff} = 12\text{mb} \\ \sqrt(s) &= 1.8\text{TeV} \qquad \sigma_{tot} = 81\text{mb} \qquad \sigma_{inel} = 50\text{mb} \qquad \sigma_{eff} = 10\text{mb}\end{aligned}$$

where σ_{eff} has been identified with $\sigma_{eff,P}$ as expressed in eq. 54.

Notice that in a single channel gaussian model the value of the effective cross section is given by $2\pi R^2$ (cfr eq.54) with R the radius of the overlap function of the matter distribution of the two hadrons, corresponding to $R^2 = 4/3\langle r^2\rangle$, where $\sqrt{\langle r^2\rangle}$ is the rms hadron radius. For the proton one might take $\sqrt{\langle r^2\rangle} = .6$ fm[7]. The effective cross section would then be of the order of 30 mb, much larger with respect to the experimental indications. In the multi-channel approach,

on the contrary, one obtains that the effective cross section results from the sum in eq.52, which leads necessarily to a smaller value with respect to the single channel case. The weights of the different channels are related to the value of the diffractive cross section. Interestingly the simplest multichannel model, able to reproduces the observed values of the diffractive cross section, leads to an effective cross section which, as discussed in the next paragraph, compares well with the experimental indications.

Effective cross section

In the analysis of double parton collisions the CDF collaboration has claimed that the measured value of the effective cross section is consistent with the result of the simplest single channel picture, where the hadron matter density is characterized by the size of the conventional rms hadron radius[6][7]. The claim is the consequence of an unfortunate mistake (in eq. 11 of ref.[7] there shouldn't be any factor 1/2) which affects the conclusions of CDF concerning the importance of parton correlations in the process, although it does not affect the actual value of the scale factor measured in the experiment.

The effective cross section quoted by CDF, $\sigma_{eff} = 14.5 \pm 1.7^{+1.7}_{-2.3}$, is nevertheless different with respect to the effective cross section discussed here above and in most of the papers on double parton scatterings. σ_{eff} has in fact a simple link with the overlap of matter distribution in the hadronic collision only when, as previously explained, the double scattering cross section is obtained from the dispersion of the distribution in the number of partonic collisions. In the analysis of CDF all events with triple parton scatterings have on the contrary been removed from the sample of events with double parton collisions. The experiment in fact has measured the *exclusive* rather than the *inclusive* double parton scattering cross section, namely it has measured the contribution to the total inelastic cross section due to double parton collisions. As in the case of CDF the fraction of events with triple scatterings is 17% of the collected sample, the difference between the two quantities is not negligible.

An indication on the actual value of the scale factor may be obtained making a few simplifying hypotheses. The experiment has measured the rate of events with three minijets and one prompt photon. One may neglect the contamination of events with four or more parton collisions. Let's call $\mathrm{P}_2^{A,B}(\beta)$ and $\mathrm{P}_3^{A,B}(\beta)$ the probabilities of double and triple parton scattering at fixed impact parameter, the two differnt parton processes being the parton collision giving a photon + a minijet (A) and two minijets (B) respectively. Let's in addition assume that triple scatterings are only due to single collisions of kind A and double collisions of kind B.
Since the contamination of the collected sample due to triple collisions is 17% one may estimate:

$$\begin{aligned}
\sigma_D^{A,B} &= \langle N_B\rangle\sigma^{A,B}_{hard} \\
&\simeq \int d^2\beta \quad \mathrm{P}_2^{A,B}(\beta) + 2\int d^2\beta \quad \mathrm{P}_3^{A,B}(\beta) \\
&= \big[\sigma_D\big]_{CDF} + 2\times\frac{17}{83}\big[\sigma_D\big]_{CDF} \approx 1.34\big[\sigma_D\big]_{CDF} \qquad (61)
\end{aligned}$$

where the factor 2 in front of $\mathrm{P}_3^{A,B}(\beta)$ is due to the multiplicity of collisions of kind B. One hence obtains

$$\sigma_{eff} = \frac{(\sigma_{eff})_{CDF}}{1.34} \approx \quad 11\mathrm{mb} \tag{62}$$

Amazingly the result is rather close to the value obtained in the multichannel eikonal model.

4. CONCLUDING REMARKS

The aim of this paper is to study the relation between multiple parton interactions and hadronic diffraction. To that purpose the multichannel eikonal model of high energy hadronic interactions has been discussed in detail, establishing an explicit relation between the two phenomena. The puzzling feature of multiparton interactions, the small value of the effective cross section, finds a natural explanation in this framework. From a qualitative point of view the reduction of the value of the effective cross section, with respect to the value obtained in a single channel model of high energy hadronic interactions, is an intrinsic feature of the multichannel approach. Interestingly, the simplest eikonal model, able to reproduce high energy hadronic diffraction, gives a value of the effective cross close to the experimental indications.

The non perturbative input of the multiparton cross section is represented by the multiparton distribution functions, which depend on the scale of the process, on the kinematical variables, on the different kinds of interacting partons and on their relative transverse distance. In a multiparton collision, the relative transverse distance is naively related to the hadron form factor, which leads however to an effective cross section too large in comparison with experiment. The main result of the present analysis is that the form factor is an average over many different hadronic configurations, which however interact with different strengths in a high energy hadronic collisions. Multiple parton interactions are more likely to take place when the hadron is in a compact configuration, which enhances relatively small transverse distances, leading to relatively small values of the effective cross section.

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